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EFFECTS OF RADIATION ON FLOW PAST AN IMPULSIVELY STARTED INFINITE VERTICAL PLATE WITH MASS TRANSFER

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The effect of radiation on the flow past an impulsively started vertical plate in the presence of mass transfer is analyzed. The fluid is a gray, absorbing–emitting, and nonscattering medium and the Rosseland approximation is used to describe the radiative heat flux in the energy equation. A rise of the velocity due to the presence of a foreign mass is observed. An increase in the Schmidt number (Sc < 1) and in the radiation parameter N leads to a decrease in the velocity. The skin friction increases due to the presence of a foreign mass when Sc < 1 and decreases at Sc = 1.

Introduction. The unsteady flow past an impulsively started horizontal infinite plate was studied by Stokes [13]. Later on, Soundalgekar presented an exact solution for the flow past an infinite vertical isothermal plate impulsively started in a viscous incompressible fluid [11]. An effect of free convection on the flow was studied. The effect of mass transfer was investigated by Soundalgekar [12]. However, these studies were confined to normal temperatures of the surrounding medium. If the temperature of the surrounding fluid is rather high, radiation effects play an important role, but this situation does exist in space technology. In these cases, it is necessary to take into account the effects of radiation and free convection. In steady flows, such studies were performed by Cess [4], Arpaci [1], Cheng and Ozisik [5], Hasegawa et al. [6], Bankston et al. [2], Hossain and Takhar [8, 9], and Hossain et al. [7]. In the case of unsteady flows, Raptis and Perdikis presented results for the flow past a uniformly accelerated vertical plate obtained with numerical solution of the governing equation [10].

However, in nature, along with the free-convection currents caused by the temperature differences, the flow can also be affected by a difference in concentration or material constitution. For example, in atmospheric flows, differences in the H_2O concentration exist which affect the flow. Flows in bodies of water are affected by the differences in the density, temperature, and concentration of dissolved material and the kind of suspended matter. Moreover, in a number of engineering applications, foreign gases are injected. Due to such a mass transfer, in many cases a reduction in the wall shear stress, the mass or heat transfer rate was observed. Usually, H_2, O_2, H_2O, CO_2, etc. are the foreign gases which are injected into the air flowing past bodies. Hence, in flow past vertical bodies, buoyancy forces arise due to both temperature and concentration differences.

The effects of radiation on the flow past an impulsively started infinite vertical plate with mass transfer using the Rosseland approximation [3] have not received the attention of researchers. An exact solution can be derived by the Laplace-transform technique, and the results should be compared with the no-radiation case. The fluid considered is a radiating and nonscattering medium. Further, the mathematical analysis will be presented for both radiation and no-radiation cases.

Mathematical Analysis. We consider the flow of an incompressible viscous radiating fluid past an impulsively started infinite vertical plate with mass transfer. The x'-axis is taken along the plate in the vertical direction and the y'-axis is normal to the plate. It is also assumed that the radiation heat flux in the x'-direction is negligible as compared to that in the y'-direction. Under the ordinary Boussinesq approximation, the flow of a radiating fluid is shown to be governed by the following system of equations:

$$\frac{\partial u'}{\partial t'} = \upsilon \frac{\partial^2 u'}{\partial y'^2} + g\beta (T' - T'_\infty) + g\beta^* (C' - C'_\infty) , \tag{1}$$

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$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y'}, \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} \tag{3}$$

under the boundary conditions

$$\begin{aligned} &t' < 0: \quad u' = 0, \quad T' = T'_\infty, \quad C' = C'_\infty \quad \text{for all } \; y'; \\ &t' \geq 0: \quad u' = u'_0, \quad T' = T'_w, \quad C' = C'_w \quad \text{at} \quad y' = 0, \\ &u' = 0, \quad T' \to T'_\infty, \quad C' \to C'_\infty \quad \text{as} \quad y' \to \infty. \end{aligned} \tag{4}$$

We assume the Rosseland approximation [3], which leads to

$$q_r = -\frac{4\sigma}{3\kappa^*}\frac{\partial T'^4}{\partial y'}. \tag{5}$$

If the temperature difference $T'—T'_\infty$ within the flow is sufficiently small, the Taylor series for T'^4 with neglect of the higher-order terms is given by a linear temperature function:

$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty. \tag{6}$$

Based on Eqs. (5) and (6), Eq. (2) reduces to

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{16\sigma T'^3_\infty}{3\rho c_p \kappa^*}\frac{\partial^2 T'}{\partial y'^2}. \tag{7}$$

We now introduce the following dimensionless quantities:

$$\begin{aligned} &y = \frac{y' u_0}{\upsilon}, \quad u = \frac{u'}{u_0}, \quad t = \frac{t' u_0}{\upsilon}, \quad \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \quad C = \frac{C' - C'_\infty}{C' - C'_w}, \quad \mathrm{Pr} = \frac{\mu c_p}{k}, \\ &\mathrm{Sc} = \frac{\upsilon}{D}, \quad \mathrm{Gr} = \frac{\upsilon g\beta\,(T'_w - T'_\infty)}{u_0^3}, \quad N = \frac{\kappa^* k}{4\sigma T'^3_\infty}, \quad \mathrm{Gc} = \frac{\upsilon g\beta^*\,(C' - C'_\infty)}{u_0^3}. \end{aligned} \tag{8}$$

Then Eqs. (1), (6), and (3) take on the form

$$\frac{\partial u}{\partial t} = \mathrm{Gr}\,\theta + \mathrm{Gc}\,C\,\frac{\partial^2 u}{\partial y^2}, \tag{9}$$

$$\mathrm{Pr}\,N\frac{\partial \theta}{\partial t} = (3N + 4)\frac{\partial^2 \theta}{\partial y^2}, \tag{10}$$

$$\mathrm{Sc}\,\frac{\partial C}{\partial t}=\frac{\partial^2 C}{\partial y^2}\,, \tag{11}$$

and the initial and boundary conditions are

$$\begin{aligned} &t' < 0: \; u=0\,,\; \theta=0\,,\; C=0 \quad \text{for all } y\,; \\ &t' \ge 0: \; u=1\,,\; \theta=1\,,\; C=1 \quad \text{at } y=0\,, \\ &u=0\,,\; \theta=0\,,\; C=0 \quad \text{as } y\to\infty\,. \end{aligned} \tag{12}$$

The solutions to Eqs. (9), (10), and (11) under various conditions are now derived by the ordinary Laplace-transform technique:

1. At $\mathrm{Sc}\neq 1$

$$u=\mathrm{erfc}\,(\eta)-\left[\frac{\mathrm{Gr}\,(3N+4)\,t}{(3-\mathrm{Pr})\,N+4}+\frac{\mathrm{Gc}}{1-\mathrm{Sc}}\right]\left[(1+2\eta^2)\,\mathrm{erfc}\,(\eta)-\frac{2\eta}{\sqrt{\pi}}\exp\,(-\eta^2)\right]+$$

$$+\frac{\mathrm{Gr}\,(3N+4)\,t}{(3-\mathrm{Pr})\,N+4}\left\{\left(1+\frac{2\mathrm{Pr}\,N\eta^2}{3N+4}\right)\mathrm{erfc}\left(\eta\sqrt{\frac{\mathrm{Pr}\,N}{3N+4}}\right)-2\,\frac{\sqrt{\mathrm{Pr}\,N}}{\sqrt{\pi}\,(3N+4)}\,\eta\exp\left(-\eta^2\,\frac{\mathrm{Pr}\,N}{3N+4}\right)\right\}+$$

$$+\frac{\mathrm{Gc}}{1-\mathrm{Sc}}\left[(1+2\eta^2\,\mathrm{Sc})\,\mathrm{erfc}\,(\sqrt{\mathrm{Sc}}\,\eta)-\frac{2\eta\sqrt{\mathrm{Sc}}}{\sqrt{\pi}}\exp\,(-\eta^2\,\mathrm{Sc})\right], \tag{13}$$

$$\theta=\mathrm{erfc}\left(\eta\sqrt{\frac{\mathrm{Pr}\,N}{3N+4}}\right), \tag{14}$$

$$C=\mathrm{erfc}\,(\eta\,\sqrt{\mathrm{Sc}})\,, \tag{15}$$

where $\eta=\dfrac{y}{2\sqrt{t}}$.

2. At Sc = 1

$$u=\mathrm{erfc}\,(\eta)-\frac{\mathrm{Gr}\,(3N+4)\,t}{(3-\mathrm{Pr})\,N+4}\left[(1+2\eta^2)\,\mathrm{erfc}\,(\eta)-\frac{2\eta}{\sqrt{\pi}}\exp\,(-\eta^2)\right]+$$

$$+\frac{\mathrm{Gr}\,(3N+4)\,t}{(3-\mathrm{Pr})\,N+4}\left\{\left(1+\frac{2\mathrm{Pr}\,N\eta^2}{3N+4}\right)\mathrm{erfc}\left(\eta\sqrt{\frac{\mathrm{Pr}\,N}{3N+4}}\right)-2\,\frac{\sqrt{\mathrm{Pr}\,N}}{\sqrt{\pi}\,(3N+4)}\,\eta\exp\left(-\eta^2\,\frac{\mathrm{Pr}\,N}{3N+4}\right)\right\}+$$

$$+\,2\,\mathrm{Gc}\,\eta t\left[\sqrt{\frac{t}{\pi}}\exp\,(-\eta^2)-\mathrm{erfc}\,(\eta)\right], \tag{16}$$

$$\theta = \mathrm{erfc}\left(\eta\sqrt{\frac{\Pr N}{3N+4}}\right), \tag{17}$$

$$C = \mathrm{erfc}\,(\eta)\,. \tag{18}$$

In the absence of radiative effects, the solutions derived by Soundalgekar [12] are as follows:
1. At $\mathrm{Sc} \neq 1$

$$u = \mathrm{erfc}\,(\eta) + \frac{\mathrm{Gr}\,t}{\Pr - 1}\left\{(1 + 2\eta^2)\,\mathrm{erfc}\,(\eta) - (1 + 2\eta^2 \Pr)\,\mathrm{erfc}\,(\eta\sqrt{\Pr}) - \frac{2\eta}{\sqrt{\pi}}\exp(-\eta^2) + \frac{2\eta\sqrt{\Pr}}{\sqrt{\pi}}\exp(-\eta^2 \Pr)\right\} +$$

$$+ \frac{\mathrm{Gc}\,t}{\mathrm{Sc} - 1}\left\{(1 + 2\eta^2)\,\mathrm{erfc}\,(\eta) - (1 + 2\eta^2 \mathrm{Sc})\,\mathrm{erfc}\,(\eta\sqrt{\mathrm{Sc}}) - \frac{2\eta}{\sqrt{\pi}}\exp(-\eta^2) + \frac{2\eta\sqrt{\mathrm{Sc}}}{\sqrt{\pi}}\exp(-\eta^2 \mathrm{Sc})\right\}, \tag{19}$$

$$\theta = \mathrm{erfc}\,(\sqrt{\Pr}\,\eta)\,, \tag{20}$$

$$C = \mathrm{erfc}\,(\sqrt{\mathrm{Sc}}\,\eta)\,. \tag{21}$$

2. At $\mathrm{Sc} = 1$

$$u = \mathrm{erfc}\,(\eta) + \frac{\mathrm{Gr}\,t}{\Pr - 1}\left\{(1 + 2\eta^2)\,\mathrm{erfc}\,(\eta) - (1 + 2\eta^2 \Pr)\,\mathrm{erfc}\,(\eta\sqrt{\Pr}) - \frac{2\eta}{\sqrt{\pi}}\exp(-\eta^2) + \frac{2\eta\sqrt{\Pr}}{\sqrt{\pi}}\exp(-\eta^2 \Pr)\right\} +$$

$$+ 2\mathrm{Gc}\,\eta t\left\{\frac{\exp(-\eta^2)}{\sqrt{\pi}} + \eta\,\mathrm{erfc}\,(-\eta)\right\}, \tag{22}$$

$$\theta = \mathrm{erfc}\,(\sqrt{\Pr}\,\eta)\,, \tag{23}$$

$$C = \mathrm{erfc}\,(\eta)\,. \tag{24}$$

Results and Discussion. The numerical values of u, θ, and c obtained from Eqs. (13)–(24) are shown in Figs. 1–5. Figures 1 and 2 present the velocity profiles for the radiation case, whereas the no-radiation case is shown in Fig. 3. It is seen from Figs. 1 and 2 that the velocity decreases with increasing radiation parameter N and Schmidt number Sc. We can conclude from Fig. 2 that, in the presence of a foreign mass, the velocity increases with Gc. Greater cooling of the plate causes a velocity decrease in the presence of a foreign mass. Here, the effects of the Prandtl number are quite pronounced. At small values of Pr and N, the velocity of the fluid increases sharply near the plate with time t. This effect is completely absent in the absence of radiation effects. As the Prandtl number increases, the velocity near the plate decreases.

Figure 4 presents the temperature profiles in the case of the presence and absence of radiation effects. It is seen that the temperature increases when N decreases, with a larger increase for the smaller Prandtl numbers.

Figure 5 shows the concentration profiles for different values of the Schmidt number. It is seen that the concentration increases with a decrease in Sc.

From the velocity field, we determine the skin friction, which in dimensionless form is given by

$$\tau_{\mathrm{w}} = \tau'/\rho U_0^2 = -\left.\frac{du}{d\eta}\right|_{\eta=0}. \tag{25}$$

Substituting Eqs. (13) and (16) in Eq. (25), we get for $\mathrm{Sc} \neq 1$

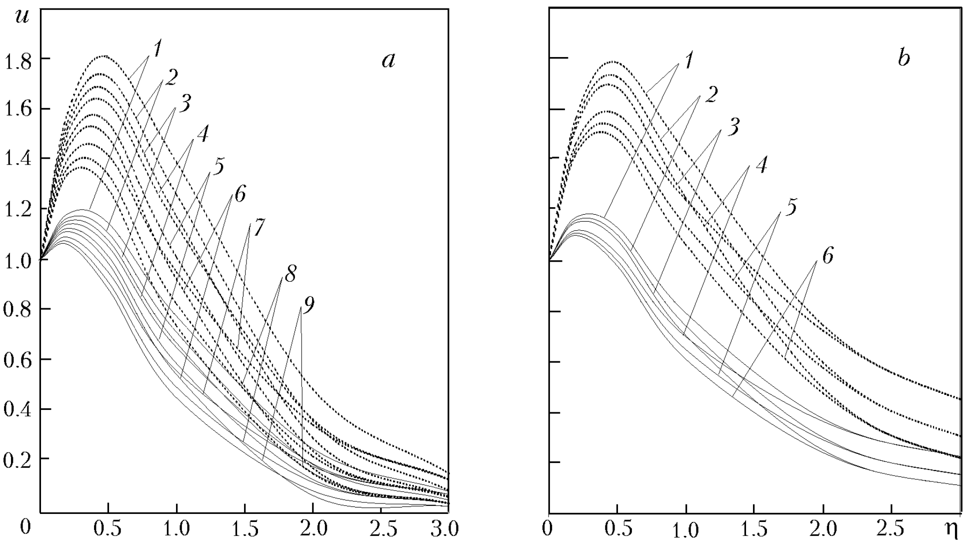


Fig. 1. Velocity profiles for various values of Sc, *N*, and *t* at Gr = Gc = 4 with Pr = 0.71 (a) and 0.2 (b): a) Sc = 0.22 and *N* = 1 (1), 3 (2), and 30 (3); Sc = 0.6 and *N* = 1 (4), 3 (5), and 30 (6); Sc = 2 and N = 1 (7), 3 (8), and 30 (9); b) Sc = 0.6 and *N* = 1 (1), 3 (2), and 30 (3); Sc = 2 and *N* = 1 (4), 3 (5), and 30 (6). Solid and dashed curves correspond to *t* = 0.2 and 0.4.

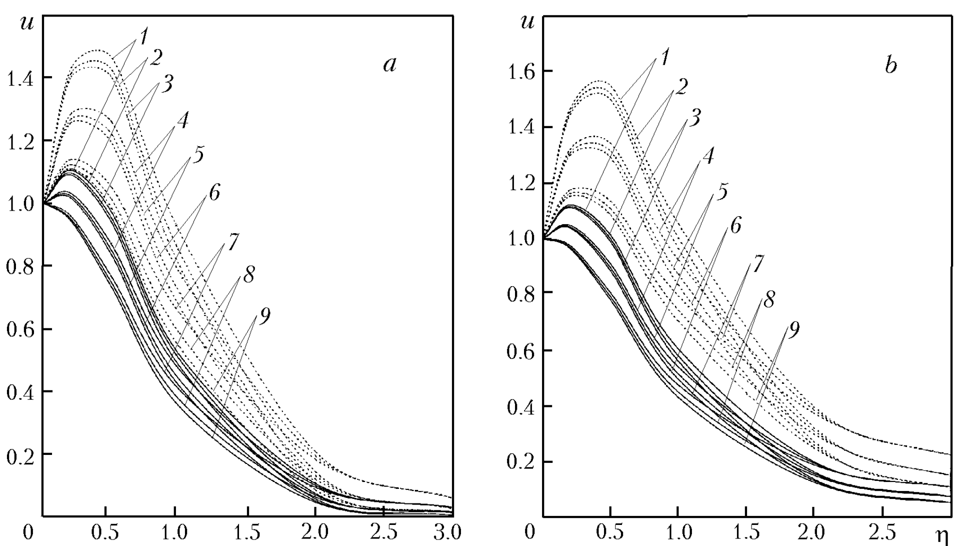
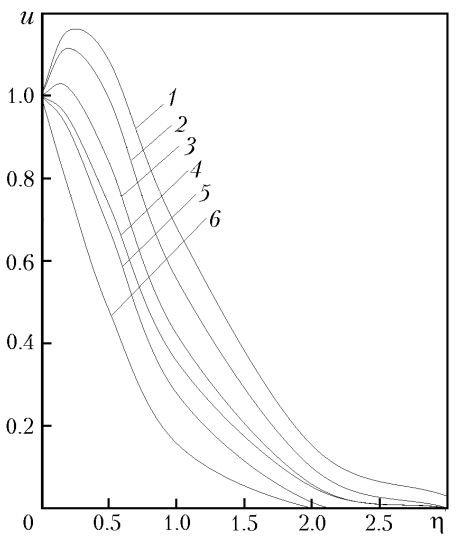


Fig. 2. Velocity profiles for various values of Gc, *N*, and *t* at Sc = 0.6 and Gr = 2 for Pr = 0.71 (a) and 0.2 (b): Gc = 6 and *N* = 1 (1), 3 (2), and 30 (3); Gc = 4 and *N* = 1 (4), 3 (5), and 30 (6); Gc = 2 and *N* = 1 (7), 3 (8), and 30 (9). Solid and dashed curves correspond to *t* = 0.2 and 0.4.

$$\tau_w = -\frac{2}{\sqrt{\pi}} + \frac{4}{\sqrt{\pi}}\left[\frac{\text{Gr}\,(3N+4)}{(3-\text{Pr})\,N+4}\left(1-\sqrt{\frac{\text{Pr}N}{3N+4}}\right)+\frac{\text{Gc}\,t}{1-\text{Sc}}\,(1-\sqrt{\text{Sc}})\right] \tag{26}$$

and for Sc =1

$$\tau_w = -\frac{2}{\sqrt{\pi}} + \frac{4}{\sqrt{\pi}}\left[\frac{\text{Gr}\,(3N+4)}{(3-\text{Pr})\,N+4}\left(1-\sqrt{\frac{\text{Pr}N}{3N+4}}\right)+2\,\text{Gc}\,t\left(\sqrt{\frac{t}{\eta}}-1\right)\right]. \tag{27}$$



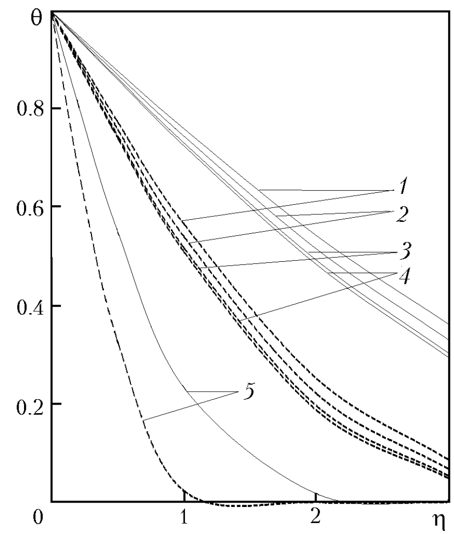


Fig. 3. Velocity profiles in the absence of radiation for various values of Sc, Gc, N, and t at Gr = 2: 1) Sc = 0.22, Gc = 2, Pr = 0.2, and t = 0.2; 2) 0.6, 2, 0.2, and 0.4; 3) 0.6, 4, 0.2, and 0.2; 4) 0.6, 2, 0.2, and 0.2; 5) 0.6, 2, 0.71, and 0.2; 6) 1, 2, 0.2, and 0.2.

Fig. 4. Temperature profiles in the presence (curves 1–4) and absence (curves 5) of radiation for Gr = Gc = 4, Sc = 0.6, t = 0.2, and Pr = 0.2 (solid curves) and 0.71 (dashed curves): 1) N = 3, 2) 5, 3) 10, 4) 15, 5) 0.

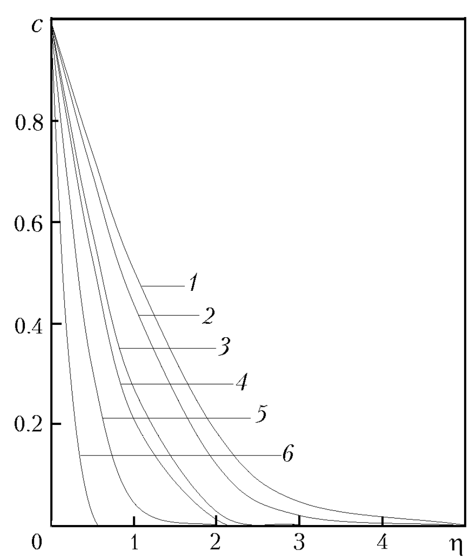


Fig. 5. Concentration profiles in the absence of radiation for various values of Sc: 1) Sc = 0.22, 2) 0.3, 3) 0.6, 4) 0.78, 5) 2, 6) 10.

In the absence of radiation, substitution of Eqs. (19) and (22) in Eq. (25) gives for Sc ≠ 1

$$\tau_w = \frac{1}{\sqrt{\pi t}} \frac{\mathrm{Gr}\, t}{\mathrm{Pr} - 1} (\sqrt{\mathrm{Pr}} - 3) + \frac{\mathrm{Gc}\, t}{\mathrm{Sc} - 1} (\sqrt{\mathrm{Sc}} - 3) - 1 \tag{28}$$

and for Sc = 1

TABLE 1. Values of the Dimensionless Skin Friction in the Presence of Radiation

Gr	Gc	Sc	N	Pr = 0.2, t = 0.2	Pr = 0.2, t = 0.4	Pr = 0.71, t = 0.2	Pr = 0.71, t = 0.4
				τ_w			
2	2	0.22	1	0.422	–1.236	0.471	–1.216
			3	0.443	–1.213	0.475	–1.286
			30	0.456	–1.205	0.461	–1.403
		0.6	1	1.994	–0.858	1.931	–1.285
			3	1.989	–0.937	1.823	–1.801
			30	1.975	–1.035	1.679	–2.44
		0.78	1	6.159	1.052	5.836	–0.416
			3	6.095	0.737	5.469	–1.968
			30	6.02	0.392	5.022	–3.817
		1	1	–1.154	–0.013	–1.242	–0.188
			3	–1.183	–0.071	–1.284	–0.272
			30	–1.206	–0.116	–1.315	–0.334
4	4	0.6	1	–0.637	–9.836	–0.763	–10.692
			3	–0.647	–9.994	–0.979	–11.723
			30	–0.674	–10.19	–1.267	–13.001

TABLE 2. Values of the Dimensionless Skin Friction in the Absence of Radiation

Gr	Gc	Sc	Pr = 0.2, t = 0.2	Pr = 0.2, t = 0.4	Pr = 0.71, t = 0.2	Pr = 0.71, t = 0.4
			τ_w			
2	2	0.22	–0.569	0.087	–0.644	–0.019
	2	0.6	–0.629	0	–0.703	–0.103
	2	0.78	–0.645	–0.02	–0.72	–0.126
	2	1	0.096	1.028	0.22	0.923
	4	0.6	–0.344	0.405	–0.419	0.3
	6	0.6	–0.06	0.808	–0.135	0.702
	4	1	1.106	2.456	1.031	2.35

$$\tau_w = \frac{1}{\sqrt{\pi t}} \frac{\mathrm{Gr}\, t}{\mathrm{Pr} - 1} (\sqrt{\mathrm{Pr}} - 3) + 2t\, \mathrm{Sc} - 1 . \tag{29}$$

The numerical values of τ_w obtained from Eqs. (26) and (27) are listed in Table 1. It is seen from this table that for Gr = Gc = 2 at t = 0.2, Pr = 0.2, and Sc = 0.22 the skin friction increases with the radiation parameter N. At Sc = 1, τ_w is found to become negative for both values of Pr and t, i.e., a reverse flow occurs. A reverse flow also takes place at Gr = Gc = 4 and Sc = 0.6.

In Table 2, numerical values of the skin friction τ_w in the absence of radiation evaluated from Eqs. (28) and (29) are presented. It is seen that at Gr = Gc = 2 and t = 0.2 skin friction becomes negative for small Sc. When Sc = 1, $\tau_w > 0$.

Conclusions. The radiation effect on the free convection flow past an impulsively started infinite vertical plate is investigated in the Rosseland diffusion approximation. The transformed dimensionless equations are solved by the Laplace-transform technique. The results are presented for the velocity, temperature, and skin friction at various values of Gr, Gc, Sc, Pr, t, and the radiation parameter N. It is observed that the velocity increases due to the presence of a foreign mass. An increase in Sc (Sc < 1) leads to a drop in the velocity, and for Sc =1, an increase in t and Gc leads to a rise in the velocity. It is also seen that an increase in the radiation parameter N leads to velocity and temperature decreases. For Gr = Gc = 2 and small values of t, Pr, and Sc, the skin friction increases with the radiation parameter N. At Sc = 1, τ_w becomes negative. In the absence of radiation, at Gr = Gc = 2 and small t and Sc skin friction is negative too.

NOTATION

C, dimensionless concentration; C', species concentration; C'_∞, species concentration at infinity; C'_w, species concentration at the plate; c_p, specific heat at constant pressure; D, diffusivity; Gc, modified Grashof number; Gr, Grashof number; g, acceleration due to gravity; k, thermal conductivity; N, radiation parameter; Pr, Prandtl number; q_r, radiative heat flux in the y'-direction; Sc, Schmidt number; T', temperature of the fluid; T'_∞, temperature of the fluid at infinity; T'_w, temperature of the plate; t', time; t, dimensionless time; u, dimensionless velocity of the fluid; u', velocity of the fluid; u_0, velocity of the plate; x' and y', coordinates along and normal to the plate; x and y, dimensionless coordinates along and normal to the plate; α, thermal diffusivity; β, volumetric coefficient of thermal expansion; β^*, volumetric coefficient of species expansion with concentration; $\eta = y/2\sqrt{t}$; κ^*, mean absorption coefficient; ν, kinematic viscosity; θ, dimensionless temperature; ρ, density; σ, Stefan–Boltzmann constant; τ, skin friction; τ_w, dimensionless skin friction. Subscripts: w, on the wall; ∞, at infinite distance.

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